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The EFFECT OF INFLUENTIAL PARAMETERS OF MOVING MASS ON DYNAMIC RESPONSE OF BEAM USING GREEN FUNCTION

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ABSTRACT

In this paper, the dynamic behavior of the uniform simply supported Bernoulli Euler beam structures subjected to moving mass is analyzed traversing its span using Green Function approach. Green Function is easily suitable for various boundary conditions as it is already embedded in its expression. The validation of this method is shown by comparing it by several reference papers. The influences of variation of travelling velocity of mass and effect of increase in magnitude of moving load is studied which have significant effect on dynamic response of simply supported beam considering inertial effects of mass. A MATLAB code was developed to compute dynamic analysis and plot the deflection results.

KEYWORDS: Dynamic analysis, Green Function, Sub-critical velocity, Mass Ratio, Bernoulli Euler, Simply supported beam.

INTRODUCTION

Dynamic response of a supported beam under a moving load is an interesting topic of great practical importance to engineering. Interest in this problem originates in Mechanical design engineering for parking garages and aircraft carriers, advanced propulsion concepts such as high-speed precision machining, magnetic disk drives and cables transporting materials and structural engineering for the design of railroad tracks, railroad bridges, and highway bridges, where the accurate calculation of loads and deflection is essential for a reliable design to avoid vibration. Because, this vibration can create cracks, poor quality jobs and can decrease the life span of the material used for different purpose. It has been observed that, as a structure is subjected to moving loads, the dynamic deflection and stresses gets significantly higher than those observed in the static case. These moving loads, with their inertial effects taken into account are called moving masses which cannot be neglected due to their moving state. Hence, the dynamic deflection estimation is of immense importance in the design of the structural components of such systems.

Mackertich [1] studied the dynamic response of simply supported beam using modal superposition method excited by a moving mass based on Timshenko beam theory with corrections for the shear deformation and rotatory inertia effects and also compared its response with that of Bernoulli Euler one. Hamada [2] used double Laplace transformation method to find a solution for a simply supported damped Bernoulli-Euler uniform beam with damping under the action of a moving mass less load with respect to both time and the length coordinate along the beam. Michaltsos [3] studied the effects of dynamic deflection of a simply supported beam including moving mass of constant magnitude and velocity using first approximation series solution. Ting et al [4] developed an algorithm to solve the classical problem of the dynamic response of a finite elastic beam supporting a moving mass including the boundary conditions and only need to define its initial conditions. He also differentiated between three different frequency regions having different characteristics with respect to each other sub critical, critical and super critical. C. W. Bert [5] presented comparative evaluation of six different numerical integration methods. He used damped, undamped, linear and nonlinear and converged solutions for comparing the accuracy of results solved for fixed smaller time steps. Numerical efficiency is measured by the total time required to calculate the system response. A.

S. Mohamed [6] determined the dynamic characteristics of Euler Bernoulli beams with attached masses and spring using Greens Function which is suitable for determining natural frequencies and mode shapes of beams with

intermediate attachments and of various boundary conditions. H. P. Lee [7] formulated equation of motion in matrix form for the dynamic response of a Timoshenko beam acted upon by a mass moving with a constant speed using the Lagrangian approach neglecting the inertial effects of mass by using Runge Kutta Method. It was found that separation of the mass from the beam may occur for a high axial speed of the mass. M. Ichikawa et al [8] investigated the behavior of the multi-span continuous beam acted upon by a moving mass at a constant velocity, which assumes that each span of the continuous beam obeys uniform Euler-Bernoulli beam theory. The solution to this system is simply obtained by taking both Eigen function expansion or the modal analysis method and the direct integration method combined. J. D. Achenbach et al [9] found solutions for uniform beam of infinite length subjected to a force with constant velocity, having solutions that are time invariant in a coordinate system. G. G. Lueschen et al [10] studied the closed form solution for the Green's functions to formulate for uniform Timoshenko beams to show that Green function for uniform Euler-Bernoulli beams, both with and without constant preaxial loads, are having matching form. P. K. Chatterjee et al [11] investigated the dynamic behavior of multi-span continuous beams under a moving load, by considering the effect of interaction between the vehicle and the bridge road, the torsion in the bridge due to eccentrically placed vehicles and the randomness of the surface irregularity of the road. Arturo O. Cifuentes [12] presented a combined finite element and finite difference technique to determine the response of a beam subjected to a moving mass based on a Lagrange Multiplier formulation to make it compatible at the beam-mass interface using a set of auxiliary functions used for any boundary conditions. G.T. Michaltsos et al [13] have studied the effect of centripetal and coriolis force on the linear dynamic response of a simply supported light (steel) bridge under a moving load-mass of constant magnitude and velocity. G. T. Michaltsos [14] deals with the linear dynamic response of a simply supported Euler Bernoulli elastic single span beam subjected to a moving one or two axle load of constant magnitude and variable velocity and focuses on the effect of the acceleration or deceleration on the behavior of the beam and also the influence of the damping of the beam. Ismail Esen [15] studied the dynamic behaviour of beam carrying accelerating moving mass that is travelling on beam modelled as finite element in order to include inertial effect beside gravitation force of mass, centripetal force and coriolis force. B. Mehri et al [16] presented the linear dynamic response of uniform Euler Bernoulli beams with different boundary conditions excited by a moving load using a dynamic green function and studied the effects of velocity of load and other parameters. Jia-Jang Wu et al [17] in his paper presents a technique for using finite element packages for analyzing the dynamic response of simply supported beam subject to single time variant moving loads and then it is applied to the problem on two-dimensional motion of the trolley of a mobile gantry crane model. M. Dehestani et al [18] In this study an analytical–numerical method is presented which can be used to determine the dynamic response of beams carrying a moving mass, with various boundary conditions to show that the Coriolis acceleration, associated with the moving mass as it traverses along the vibrating beam shall also be considered and Critical influential speeds was introduced. M. Foda et al [19] used Dynamic Green Function approach to determine the response of simply supported Bernoulli Euler beam subject to moving mass traversing its span with and without inertial effects. I. O.Abiala [20] obtained the dynamic response of beam under uniformly distributed moving loads by using Newmarks integration and finite element model by applying Galerkins Weighted Residual Method in order to obtain the effect of velocity and loads length on response of beam. B. Ryu et al [21] studied the dynamic response of beam caused by moving mass in investigated by Runge Kutta integration and in order to suppress the vibration of beam generated by moving mass using fuzzy control. M Abu-Hilal [22] determined forced vibration of damped, single, multi-span and multi loaded Euler Bernoulli beam subjected to concentrated and distributed loads with the use of Green Function.

In this paper a Green functions approach is been used for determining the dynamic response of Euler– Bernoulli beams subjected to moving mass. Green functions Method is more efficient than the series methods because it yields exact solutions in closed forms which are in particular more essential for calculating dynamic stresses and determining the dynamic response of beams other than simply supported ones. Also with the use of the Green functions method, the boundary conditions are embedded in its expression of the corresponding beams. Furthermore, by using this method, it is not necessary to solve the free vibration problem in order to obtain the eigen values and the corresponding eigen functions which are required while using series solutions and In this paper computation of dynamic response of beam is done by using MATLAB software to see the influence of variations of velocity and mass ratio on dynamic response of beam and introduced a term sub critical velocity after which deflection decreases as the velocity increase further.

THEORY AND FORMULATING THE SOLUTION

For moving mass problem, the Bernoulli Euler equation for non uniform beam of finite length, subject to a concentrated force [19] is

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + m\frac{\partial^2 w(x,t)}{\partial t^2} = F\delta(x-u), \quad (1)$$

where E is Young's modulus, I is the second moment of area of the beam cross-section, m is the mass per unit length, x is the axial co-ordinate, t is the time, $w(x, t)$ is the transverse deflection of the beam, F is the applied force and $\delta(x\text{-}u)$ is the Dirac delta function.

The boundary conditions for a simply supported beam and the initial conditions are

$$w(x,t) = \frac{\partial^2 w(x,t)}{\partial t^2} = 0 \text{ at } x=0 \text{ and } L \quad (2)$$

$$w(x,0) = \frac{\partial w(x,0)}{\partial t} = 0 \quad (3)$$

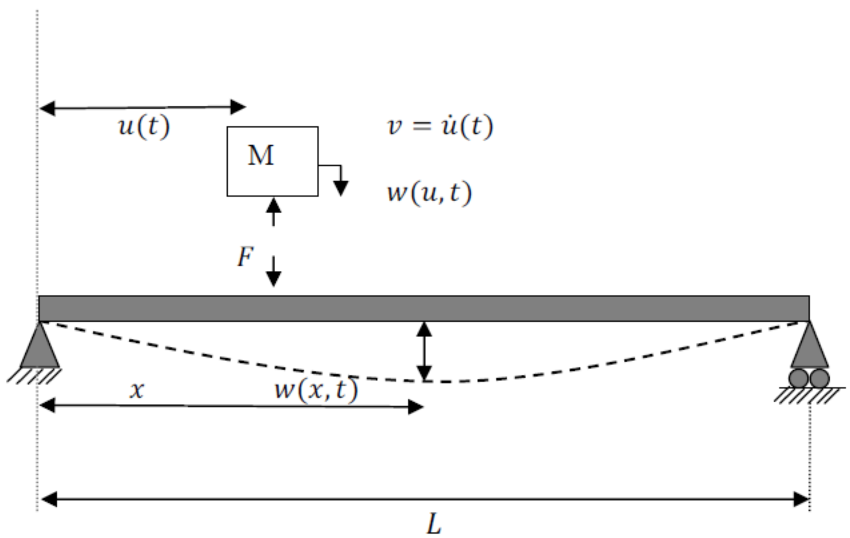

Fig.1 A mass travesing a beam with constant velocity

According to Newtons second Law of motion

$$F = M\left(g - \frac{d^2 w(x,t)}{dt^2}\right) \quad (4)$$

Where, F is the reaction force exerted by the mass M on the beam, g is the gravitational acceleration and $w(x, t)$ is the transverse displacement of the mass.

The influence function used is Green function with time variance. Hence Dynamic green Function is utilized to find the solution for the differential equation (1). Hence, if $G(x,u)$ is the dynamic Green function, which is to be calculated, for the stated problem, then the solution of equation (1) takes the form

$$w(x,t) = G(x,u)F \tag{5}$$

$$\frac{d^4W(x)}{dx^4} - q^4 w(x) = \delta(x-u) \tag{6}$$

Where q is the frequency parameter (separation constant) and is given by

$$q^4 = w^2 m/EI \tag{7}$$

In which *w* is the circular frequency that expresses the motion of the mass and is equal to $\pi v/L$.
The solution of equation (6) the Greens functions for beam of generalized boundary conditions evaluated with standard procedure given in equation (13) in references [6] is given by

$$G(x,u) = \{A_1\cos(qx) + A_2\sin(qx) + A_3\cosh(qx) + A_4\sinh(qx)\} \qquad 0 \le x \le u$$
$$G(x,u) = \{B_1\cos(qx) + B_2\sin(qx) + B_3\cosh(qx) + B_4\sinh(qx)\} \qquad x \le u \le L \tag{8}$$

The eight constants $A_1, \ldots\ldots, A_4$ and $B_1, \ldots\ldots, B_4$ are evaluated such that the Green function *G(x,u)*satisfies the following conditions

(a) Two boundary conditions at each end of the beam depending on the type of end support – for a simply supported beam,

$$G(0,u) = G(L,u) = G''(0,u) = G''(L,u) \tag{9}$$

Where, the prime indicates a derivative with respect to *x*.

(b) Continuity conditions of displacement slope and moment at *x=u*, i.e.

$$G(u^+,u) = G(u^-,u), \quad G'^{(u^+,u)} = G'^{(u^-,u)}, G''(u^+,u) = G''(u^-,u); \tag{10}$$

(c) Shear force discontinuity of magnitude one at *x=u*, i.e,

$$EI\left[G'''^{(u^+,u)} - G'''^{(u^-,u)}\right] = 1 \tag{11}$$

The Green function determined by the procedure [6] is given by

$$G(x,u) = \frac{1}{2EIq^3\sin(qL)\sinh(qL)}\{g(x,u) 0 \le x \le u \text{ and}$$
$$\{g(u,x); \quad x \le u \le L \tag{12a}$$

Where,

$$g(x,u) = \sinh(qL)\sin(qx)\sin(qL-qu) - \sin(qL)\,sinh(qx)\sinh(qx)\sinh(qL-qu) \tag{12b}$$

According to Maxwell-Rayleigh reciprocity law, *g(u,x)* is obtained by switching *x* and *u* in *g(x,u)*, which follows from the fact that *G(x,u)* must be symmetric.

The expression given by equation (12) reduces to static Green function (beam influence coefficients), when q equals to zero and is given by

$$\lim_{q \xrightarrow{yields} 0} G(x,u) = \frac{L^3}{6EI}\left(1-\frac{u}{L}\right)\frac{x}{L}\left[2-\left(\frac{x}{L}\right)^2-\left(\frac{u}{L}\right)^2\right], 0 \le x \le u \tag{13}$$

According to references [4] to change the variable by using the relationship *u=u(t)*, so that

$$\frac{d\beta}{dt} = \frac{d\beta(t)}{du}\frac{du}{dt} = v\frac{d\beta(u)}{du}$$
$$\frac{d^2\beta}{dt^2} = v^2\frac{d^2\beta(u)}{du^2} \tag{14}$$

Where,

$$\beta(u) = w(x,u) \text{ at } x=u \tag{15}$$

Eliminating F between equations (4) and (6) and making use of equation (15) yields

$$w(x,u) = G(x,u)M\left[g - v^2\frac{d^2\beta(u)}{du^2}\right] \quad (16)$$

Equation (16) is a second order differential equation, denoting the dynamic beam deflection at position x caused by the mass at location u. The boundary conditions, equations (2), are embedded in the Green function. According to references [2] one should add the complementary solution $w_d(x,u)$which is given by

$$w_d(x,u) = \frac{-2MgL^3}{\pi^4 EI}\sum_{j=1}^{\infty}\frac{\alpha}{j^3(j^2-\alpha^2)}sin\left(\frac{j\pi x}{L}\right)sin\left(\frac{j^2\pi u}{\alpha L}\right), \qquad 0 \le x, u \le \mathrm{L}, \quad (17a)$$

Where, the speed parameter α is defined as

$$\alpha = \frac{vL}{\pi}\sqrt{\frac{m}{EI}} \quad (17b)$$

It is to be noted that for a moving force problem, in which the inertial term of the moving mass is removed, the forced vibration part of the deflection as is given by equation (16) is

$$w_f(x,u) = \frac{Mg}{2EIq^3\sin(qL)\sinh(qL)}\begin{cases} g(x,u), & 0 \le x \le u \text{ and} \\ g(u,x), & x \le u \le \mathrm{L} \end{cases} \quad (18)$$

According to reference [19], the Fourier series solution of the moving force at a non critical speed is

$$W(x,u) = W_d(x,u) + w_f(x,u) \quad 0 \le x,u \le L \quad (19)$$

Where,

$$w_f(x,u) = \frac{2MgL^3}{\pi^4 EI}\sum_{j=1}^{\infty}\frac{1}{j^2(j^2-\alpha^2)}sin\left(\frac{j\pi x}{L}\right)sin\left(\frac{j\pi u}{L}\right), \quad 0 \le x,u \le L \quad (20)$$

Thus, by making use of the dynamic Green function, the sum of the Fourier series, equation (20) has been obtained in a closed form.

Therefore, equations (18) and (20) are different representations for the forced vibration part of the deflection among which equation (18) is maximum deflection at a point of a beam without inertial mass whereas, equation (20) is deflection with inertial mass. Verification of this result may be obtained by evaluating these two expressions numerically, after recalling the fact that $q = \pi\alpha^{1/2}$ and truncatingthe series solution given by equation (20) after 12 terms.

Equation (18 and 20) may be placed into non-dimensional form so that the numerical results presented are applicable for large combinations of system parameters. This is achieved by lettingw_{st}be the scaling factor for the transverse displacement, where $w_{st}$is the static deflection at the beam mid-span due to the weight of the mass, and by letting *T* be the time scale. Where, *T* is the period of the lowest vibration mode of the beam. Thus

$$w_{st} = \frac{MgL^3}{48EI}, \quad T = \frac{2L^2}{\pi}\sqrt{\frac{m}{EI}}. \quad (21)$$

The appropriate non dimensional quantities can be

$$\widehat{w} = \frac{w}{w_{st}}, \hat{G} = \frac{EI}{L^3}G \quad (22)$$

The non dimensional parameter γ depends on the mass ratio M/mL, the speed ratio, as given by equation (17b), $\alpha = v/v_{cr}$ and the number of segments and is given by

$$\gamma = \pi^2\left(\frac{M}{mL}\right)\left(\frac{l}{h}\right)^2\left(\frac{v}{v_{cr}}\right)^2 \quad (23)$$

Where, the critical speed v_{cr} is defined as

$$v_{cr} = 2L/T = \frac{\pi}{L}\sqrt{\frac{EI}{m}} \quad (24)$$

The case $v/v_{cr} = 1$ corresponds to resonance with the fundamental mode when the load is a constant force.

RESULTS AND DISCUSSION

Before discussing the numerical results, the formulation developed herein is validated with available analytical solution for a beam with simply supported boundary conditions acted upon by moving mass of different papers. At First a comparison of present results is matched with those obtained in Foda & Abduljabbar [19], where Figure 6 of paper 23 with the system parameters taken from same paper along with velocity ***25, 50 & 100 m/s*** is matched by using same MATLAB coding of present method and found excellent agreement between the present and those obtained in reference [19] which is also computed by same Green Function Method for mid-span deflection as shown in Fig.2.

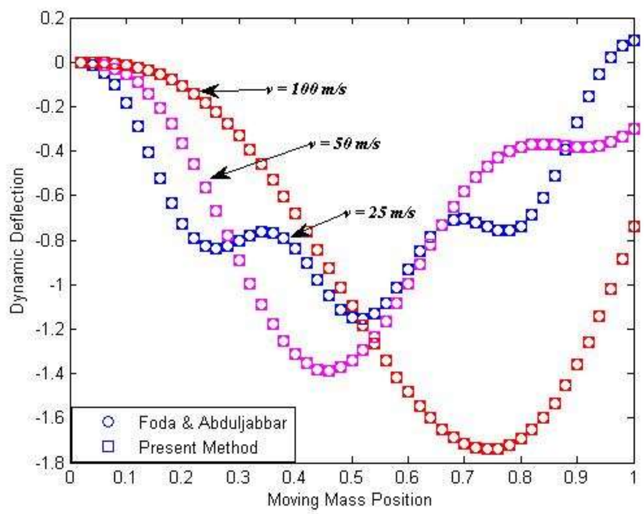

Fig.2 Comparison of Results of Present one with Foda & Abduljabbar for simply supported beam.

At second a comparison of MehriDavar & Rahmani [16] with the present method is done by using Equation No (9) along with the boundary conditions denoted by equation (10) with the following system parameters are presented taken from reference [16] and is demonstrated in Fig.3.

L = 20, v = 35 m/s, EI = 1960000000 N/m2, m = 1000 kg/m.

The vertical axis in Fig.3 shows the dimensionless mid-span deflections of the point under moving load and horizontal axis depicts the position of the load along the beam. Results of Mehri, Davar and Rahmani are computed by same Green Function approach. As can be observed in Fig.3, there is an close agreement between the two results. The dimensionless deflection is the transverse deflection at the beam mid span when a concentrated load with amplitude P is applied statically at the beams mid span.

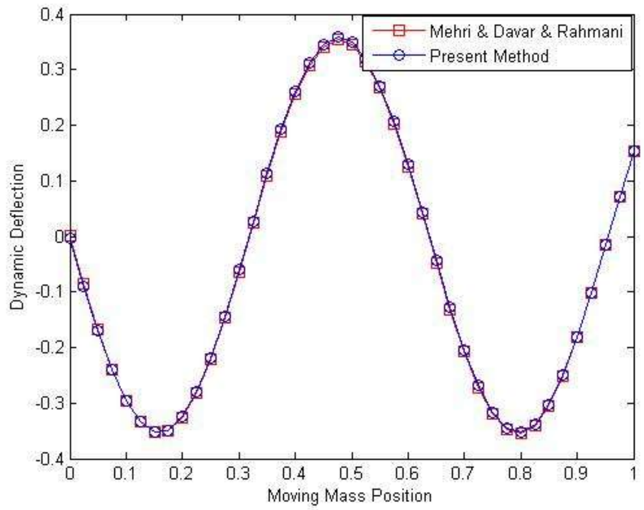

Fig.3 Comparison of Results of Present method with Mehri, Davar & Rahmani for simply supported beam.

Numerical Examples:-

In order to produce numerical analysis results of dynamic response of a beam traversed by a moving mass, computing mid-span deflection by using Green Function Method applied to Equation (21) with the data used from reference [1] a 50 *m* long two node simply supported structural beam bridge discretised into 50 uniform finite elements with assumption made of homogeneous materials. In addition, the mass per unit length *m=4800 kg/m* excited by moving mass *M=50,000kg* travelling with different velocity.

A) Effect of velocity on dynamic response of simply supported beam.

The effect of increase in velocity on dynamic response of a simply supported beam under constant load moving over the beam having Mass Ratio *0.2083* and Critical Velocity 169.2 *m/s*is shown in Fig.4. It shows mass moving with velocity *50, 75, 100, 120 and 180m/s* having maximum mid-span deflection *0.0508, 0.0607, 0.0638, 0.0633* and*0.0547* at mass position*0.46, 0.62, 0.74, 0.82* and *1* respectively, that is as the velocity increases of the moving mass, the maximum deflection of beam at mid-span position shifts towards right and also the deflection reverses its pattern of increasing long before the critical velocity *169.2m/s*, beyond 62% of that of critical velocity i.e 105 *m/s*, deflection starts decreasing, other than the observations [20,21] where, as the velocity increase than critical velocity the deflection reverses its pattern that is starts decreasing deflection only after critical velocity. The implication as per our method is that before exceeding the critical value around 62% of that of critical value of the velocity, the deflections starts decreasing, as the velocity increases which corresponds to sub-critical velocity.

In second example Fig.5 the moving mass moving with velocity *50, 125, 135, 180* and *210m/s* having maximum mid-span deflection value *0.0447, 0.0639, 0.0638, 0.0602* and *0.0564* at mass position *0.38, 0.76, 0.78, 0.92* and *1* respectively with Mass Ratio *0.3125* and critical Velocity *207.2m/s*, the deflection starts decreasing after velocity greater than *128.4m/s*.

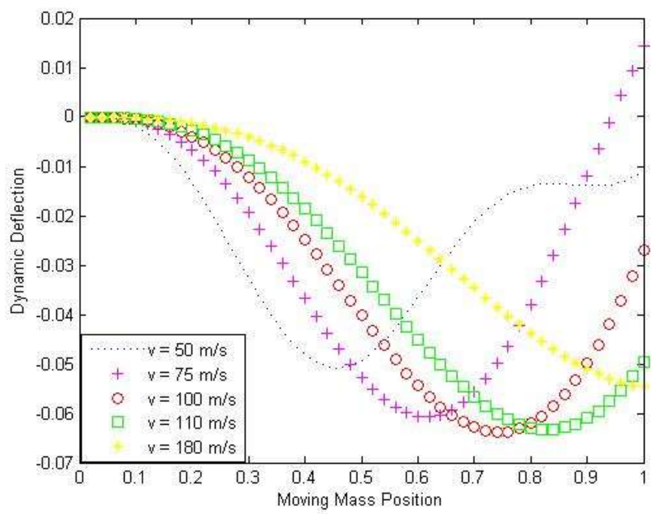

Fig.4 Dynamic response of beam for Mass Ratio=0.2083 with different velocities.

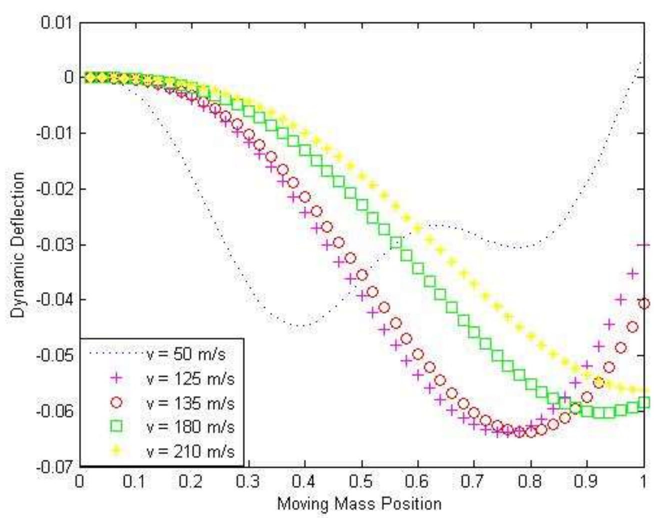

Fig.5 Dynamic response of beam for Mass Ratio=0.3125 with different velocities.

B) Effect of change in Mass Ratio on simply supported beam.

Fig.6, 7 shows the maximum mid-span dynamic deflection of the moving mass for various mass ratios γ= *0.2, 0.4, 0.6* and *1.0* when the velocity of moving mass *v= 50, 110 m/s*. When the velocity of moving mass is slow *v= 50m/s*, the maximum dynamic mid-span deflection increases as the mass ratio increases as shown in Fig.6 and its moving mass position for maximum deflection also shifts to right with the increase in velocity. The

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maximum dynamic deflection for mass ratios γ= *0.2, 0.4, 0.6, 1.0* along with velocity *v= 50m/s* are *0.0050, 0.0100, 0.0149* & *0.0249* respectively at mass position 0.46 as shown in Fig.6. When *v= 110 m/s* are *0.0062,*

0.0125, 0.0187 & *0.0312* at mass position *0.78* i.e the moving mass position shifts to the right and its maximum mid-span deflection also increases with increase in mass ratio as shown in Fig.7.

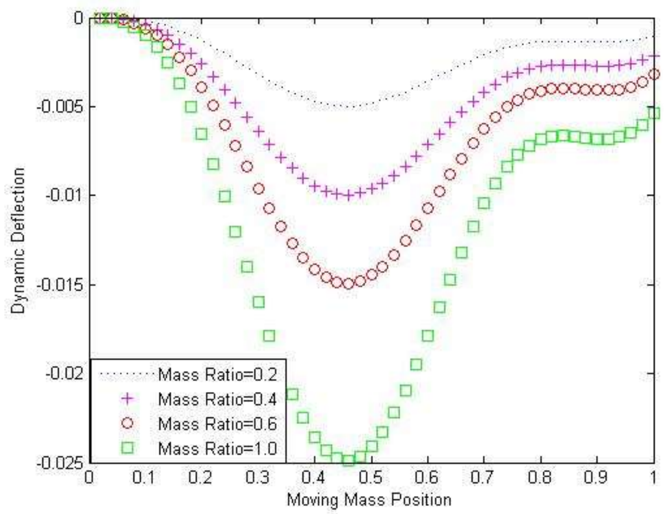

Fig.6 Dynamic response of beam for Velocity=50m/s with different Mass Ratios.

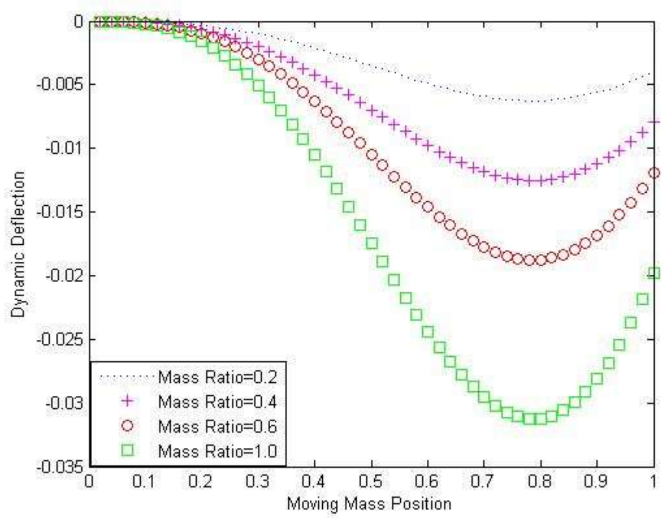

Fig.7 Dynamic response of beam for Velocity=110m/s with different Mass Ratios.

C) Effect of change in beam material on simply supported beam.

Fig.8, 9 shows the mid-span dynamic deflection of a moving mass having mass ratio *0.2083* of a simply supported beam having beam material either aluminium or steel, according to which its material strength varies and hence it affects its dynamic deflection. In Fig.8 its shows that if aluminium is used as a beam material *E=69 GPa*, its critical velocity becomes *243.17m/s* and its deflection becomes *0.0193, 0.0287, 0.0309, 0.0307* and *0.0271* at mass position *0.34*, *0.58*, *0.76*, *0.82* and *1* with velocity *50*, *100*, *150*, *160* and *250m/s* respectively. If steel beam material is used *E=200GPa*, its critical velocity becomes 414 *m/s* having deflection *0.0075*, *0.0104*, *0.0107*, *0.0106* and *0.0094* at mass position *0.4, 0.66, 0.740.8* and *1* with velocity *100*, *200*, *250*, *280* and *420m/s* respectively and comparing from both the case it is seen that with the increase in material strength the maximum mid-span deflection decreases.

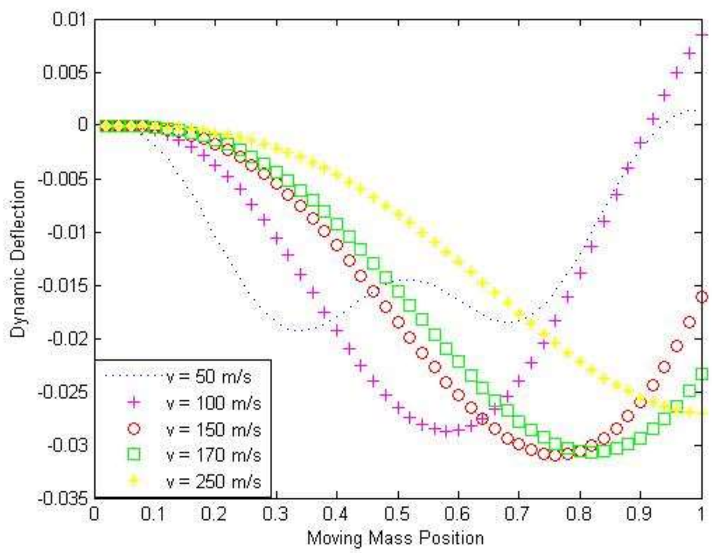

Fig.8 Dynamic response of beam for Mass Ratio=0.2083& Aluminium beam material with different velocities.

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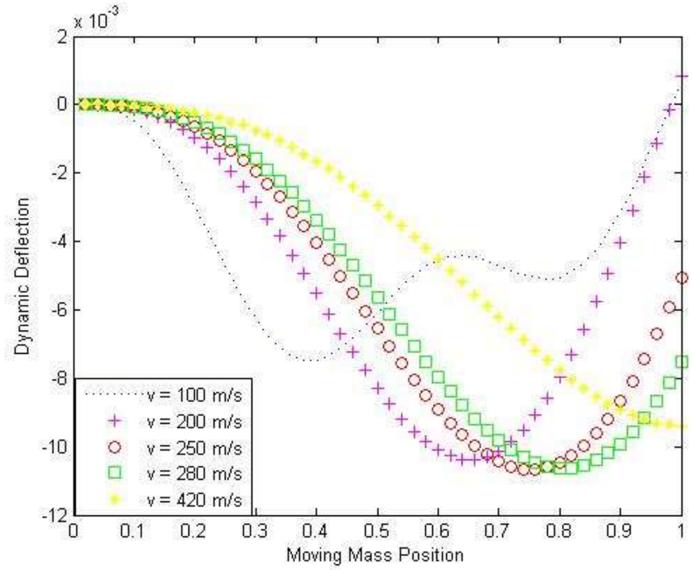

Fig.9 Dynamic response of beam for Mass Ratio=0.2083& Steel beam material with different velocities.

CONCLUSION

In this paper a method to determine the dynamic response of uniform Bernoulli Euler beam traversed by a mass moving is presented by using Green Function approach which yields exact solution and also have boundary conditions embedded in its own expression. The influences of variation of velocity and mass ratio have significant effect on dynamic mid-span deflection with inertial effect. This method was validated by comparing its result with those available in literature and found in close agreement with them.

It was concluded that as the velocity of moving mass increases the dynamic mid-span maximum deflection also increases but as the velocity goes beyond sub-critical velocity, (62% of that critical velocity) dynamic mid-span maximum deflection starts decreasing with its maximum deflection mass position shifting towards right till it reaches to the end of beam. The effect of increase in mass ratio is increase in maximum mid-span deflection with its mass position shifting towards right and the effect of increase in material strength decreases maximum mid-span deflection.

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